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# Towards dynamical Cooper pair condensation

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# 1 Introduction

Cooper [1] has shown that if interactions between electrons in a metal are attractive for the levels in the vicinity of Fermi surface, then pair of them with opposite momenta and spin create a bound state with negative energy. Since then there have been a lot of works exploring properties of the ground state for fermions with interactions. On this ground is based the consistent theory of superconductivity of Bardeen, Cooper and Schrieffer [2, 3] and Bogoliubov [4]. Works of Ginzburg and Landau allow to describe phenomenological theory of superconductive phase [5, 6], called as $\Psi$-theory.

Gorkov's work [7] is based on the following physical concept. He has shown that transition to superconductive phase can be considered as condensation of Copper's pairs. As was mentioned above, Cooper have just shown the possibility of such a pair formation [1]. Gorkov has shown [7], that analytically this phenomena is equivalent to appearance of anomalous Green's functions $\langle T\psi(x_1)\psi(x_2)\rangle$ and $\langle T\psi(x_1)\psi(x_2)\rangle$ (4.2). Thus, non-diagonal long-range order characterizes the superconductive phase transition. Complex order parameter in this theory is the anomalous Green's function with coinciding time arguments (3.7). Further Gorkov has proved that this order parameter satisfies the Ginzburg-Landau equation with the corrected charge of electrons from $e$ to the charge of Cooper's pairs $2e$ [8].

However the question of the exact process of rearrangement of the ground state in time is still open. Dilute fermionic alkali gases cooled below degeneracy [9] are expected to host the paired BCS state [10, 11]. One of the attractive features of this system is the control of the interaction strength achieved by using magnetically tuned Feshbach resonances [12, 13, 14, 15, 16, 17, 18], which provides access to the strong coupling BCS regime. Also, since the characteristic energy scales in atomic vapors are relatively low, while coherence times are long, one can perform timeresolved measurements on the intrinsic microscopic time scales, and explore a range of fundamentally new phenomena in the time dynamics of the paired state.

In this work we calculate the time evolution of "normal and anomalous“ expectation values of the creation and annihilation operators: $\langle a^+a\rangle$ and $\langle aa\rangle$, respectively, in the first and second order in $g$. Our initial motivation was to write the joint system of kinetic equations for these quantities and use it to help understanding the dynamycs of the transition to the superconducting phase, but equations appeared to be more complicated than we expected even in the first order in $g$. So we are only on the start of our work. Namely, equations are written, while their non-trivial solutions are not yet found. So far we have just shown that stationary condensate solves them.

Analogous kinetic equations appeared in the De-Sitter space as shown in [21] and [22].And analogous phenomena of condensation shown in [19] and [20].

## 1.1 Functional integral representation

In this section we review the superconductivity phenomenon in the stationary stage the with help of the function integral. Our main goal is to derive the Ginzburg-Landau (GL) functional and to calculate the thermodynamic properties of superconductors. Our discussion starts from the partition function for the BKS model:

$$Z = \int \mathcal{D}[\bar{\Psi},\, \Psi] \exp\left[-\int_0^\beta d\tau \int d^3r \bar{\Psi}_\alpha \left(\partial_\tau + \frac{1}{2m}\partial\right)\Psi_\alpha - g\bar{\Psi}_\uparrow\bar{\Psi}_\downarrow\Psi_\downarrow\Psi_\uparrow\right]. \tag{1.1}$$

In this path integral we consider fermionic fields as Grassmanian and assume non-zero temperature. Then we decouple the interaction term with the help of the Hubbard-Stratanovich transformation. NAmely, we introduce new complex scalar field $\Delta$ as follows:

$$\begin{aligned} &\exp\left(g\int_0^\beta d\tau \int d^3r\, \bar{\Psi}_\uparrow\bar{\Psi}_\downarrow\Psi_\downarrow\Psi_\uparrow\right) = \\ &= \int \mathcal{D}[\bar{\Delta},\Delta] \exp\left[-\int_0^\beta d\tau \int d^3r \left(\frac{|\Delta|^2}{g} - \left(\bar{\Delta}\Psi_\downarrow\Psi_\uparrow + \Delta\bar{\Psi}_\downarrow\bar{\Psi}_\uparrow\right)\right)\right], \end{aligned} \tag{1.2}$$

where we assume that $\Delta = \Delta(\tau, \mathbf{r})$. The next step is to write down the partition function in a compact form. To do this we introduce the following *Nambu spinors*:

$$\bar{\Phi} = \begin{pmatrix}\bar{\Psi}_\uparrow & \Psi_\downarrow\end{pmatrix}, \quad \Phi = \begin{pmatrix}\Psi_\uparrow \\ \bar{\Psi}_\downarrow\end{pmatrix}, \tag{1.3}$$

to obtain that

$$Z = \int \mathcal{D}[\bar{\Phi},\Phi] \int \mathcal{D}[\bar{\Delta},\Delta] \exp\left[-\int_0^\beta d\tau \int d^3r \left(\frac{|\Delta|^2}{g} - \bar{\Phi}\mathcal{G}^{-1}\Phi\right)\right], \tag{1.4}$$

where the matrix $\mathcal{G}^{-1}$ is as follows:

$$\mathcal{G}^{-1} = \begin{pmatrix} -\partial_\tau + \partial^2/(2m) + \mu & \Delta \\ \bar{\Delta} & -\partial_\tau - \partial^2/(2m) - \mu \end{pmatrix}. \tag{1.5}$$

In deriving (1.4) we use the following properties of Grassmannian fields:

$$\int d\tau \psi \partial_\tau \bar{\psi} = \psi\bar{\psi} - \int d\tau (\partial_\tau \psi)\bar{\psi}, \quad \text{hence} \quad \int d\tau\, \psi \partial_\tau \bar{\psi} = + \int d\tau\, \bar{\psi} \partial_\tau \psi, \tag{1.6}$$

and similar property for $\partial^2$-derivative (integrating by parts twice). Diagonal elements of (1.5) correspond to particle ,$[\mathcal{G}_0]_{11}$, and hole ,$[\mathcal{G}_0]_{22}$, Green functions. Integrating out fermionic degrees of freedom in (1.4), we obtain the following effective action for the theory under consideration:

$$S_{\text{eff}} = \int d\tau d^3 r \left( \frac{|\Delta|^2}{g} + \ln \det \mathcal{G}^{-1} \right). \tag{1.7}$$

Let us find equations of motion for this effective action. Partition function of the theory is now represented as (up to a numerical coefficient of proportionality):

$$Z = \int \mathcal{D}[\bar{\Delta}, \Delta] \exp \left[ - \int_{)}^{\beta} d\tau \int d^3 r \left( \frac{|\Delta|^2}{g} + \ln \det \mathcal{G}^{-1} \right) \right] \tag{1.8}$$

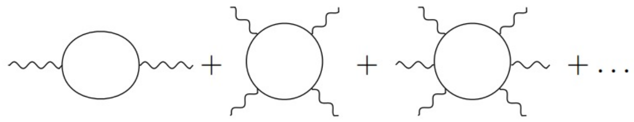
Let us rewrite the effective action as:

$$\operatorname{tr} \ln \mathcal{G}^{-1} = \operatorname{tr} \ln \left[ \mathcal{G}_0^{-1} (1 + \mathcal{G}_0 \Delta \right] = \operatorname{tr} \ln \mathcal{G}_0^{-1} - \sum_{n=0}^{\infty} \frac{1}{2n} \operatorname{tr} \left( \mathcal{G}_0 \Delta \right)^{2n},$$

$$\mathcal{G}_0^{-1} = \begin{pmatrix} -\partial_\tau + \partial^2/(2m) + \mu & 0 \\ 0 & -\partial_\tau - \partial^2/(2m) - \mu \end{pmatrix}. \tag{1.9}$$

where odd terms vanish due to zero trace. It is clear that the term $\operatorname{tr} \ln \mathcal{G}_0^{-1}$ represents the energy of free electron gas and the difference $\operatorname{tr} \ln \mathcal{G}^{-1} - \operatorname{tr} \ln \mathcal{G}_0^{-1}$ on the right hand side of the expansion (1.9) is the obtain difference in free energy between superconductor and normal metal. Graphical representation of $\ln \det(1 + \mathcal{G}_0 \Delta)$ expansion is:

where each solid line corresponds to $\mathcal{G}_0$ and each wavy line corresponds to the scalar field $\Delta$. Notice that we do not distinguish the particle and hole Green functions. Further we will see that $\Delta$ represents



the condensate up to numerical factor. Formally, it is exact transformation but further we will truncate this series and try to proof the validity of this approximation. Let us consider the first term of this expansion:

$$-\frac{1}{2}\operatorname{tr}(\mathcal{G}_0\Delta)^2 = -\sum_{q,p}[\mathcal{G}_{0,p}]_{11}[\mathcal{G}_{0,p-q}]_{22}|\Delta(q)|^2 =$$
$$= -T\sum_q\sum_m\int_p G(-i\omega_m+i\omega_n,-p+q)G(i\omega_m,p)|\Delta(q)|^2, \quad (1.10)$$

where we have used property that hole Green function corresponds to particle Green function with opposite arguments. This term can be combined with the term $|\Delta|^2/g$ and we see that in 2-nd order in $\Delta$ we have

$$S^{(2)} = \sum_q \Gamma(i\omega_n,q)|\Delta(q)|^2, \quad (1.11)$$

where $\sum_q$ denotes integration over $\mathbf{q}$ and the summation over Matrsubara frequencies.

We consider the case of non-zero temperatures. The Dyson equation for vertex function is

$$\Gamma(i\omega_n,q) = g\left(1 - gT\sum_m\int_p G(-i\omega_m+i\omega_n,-p+q)G(i\omega_m,p)\right)^{-1}, \quad (1.12)$$

where one can see the new pole appears. We perform integration over $\vec{(p)}$ and the summation over Matsubara fermionic frequenices. Thus we obtain for the second term in (1.12):

$$T\sum_m\int_p G(-i\omega_m+i\omega_n,-p+q)G(i\omega_m,p) = T\sum_m\int_p \frac{1}{(-i\omega_m+i\omega_n-\xi_{-p+q})(i\omega_m-\xi_p)}. \quad (1.13)$$

The integrand of this expression can be rewritten as

$$\frac{1}{(-i\omega_m + i\omega_n - \xi_{-p+q})(i\omega_m - \xi_p)} = \\ = \frac{1}{i\omega_n - \xi_{-p+q} - \xi_p}\left(\frac{1}{i\omega_m - \xi_p} + \frac{1}{-i\omega_m + i\omega_n - \xi_{-p+q}}\right). \quad (1.14)$$

Then, we can perform the summation over $m$ for each term separately. Let us start from the first term:

$$T\sum_m \frac{1}{i\omega_m - \xi_p} = T\sum_m \frac{-i\omega_m - \xi_p}{\omega_m^2 + \xi_p^2} = -T\sum_m \frac{\xi_p}{\omega_m^2 + \xi_p^2}, \quad (1.15)$$

where we neglect imaginary contribution due the fact that $\omega_m$ is odd (as it should be for fermions). Now we obtain the sum

$$-\frac{\xi_p}{\beta}\sum_m \frac{1}{\omega_m^2 + \xi_p^2} = \frac{1}{\beta}\int_{\mathcal{C}} \frac{dz}{2\pi i}\frac{1}{(\beta z)^2 - \xi_p^2}\frac{\beta}{2}\tanh\frac{\beta z}{2} = \\ = +\frac{\xi_p}{2}\frac{1}{\xi_p}\tanh\frac{\beta\xi_p}{2} = \frac{1}{2} - n_F(\xi_p). \quad (1.16)$$

For the second term one can denote that shifting of $m \to m - n$ removes $n$-dependence and then we have the similar summation but now we obtain $1/2 - n_F(-\xi_{-p+q})$ (it is clear from the structure of the ratio). Finally, we obtain

$$T\sum_m \int_p \frac{1}{(-i\omega_m + i\omega_n - \xi_{-p+q})(i\omega_m - \xi_p)} = \int_p \frac{1 - n_F(\xi_p) - n_F(\xi_{-p+q})}{i\omega_n - \xi_p - \xi_{-p+q}} = \chi(\omega_n, q), \quad (1.17)$$

where we have used that $n_F(-E) = 1 - n_F(E)$. We also can show the appearance of the pole in the vertex function, signalizing the new bound state, for that consider static and spatially homogeneous situation which is defined by $\omega_n = 0, \quad q = 0$.This assumption reduces our integral from (1.17) to the following:

$$\int_{-\omega_D}^{\omega_D} d\epsilon\nu(\epsilon)\frac{1 - 2n_F(\epsilon)}{2\epsilon} \approx \nu_F \ln\left(\frac{\omega_D}{T}\right). \quad (1.18)$$

Therefore, we see that the pole in vertex function appears when

$$T = T_c = \omega_D \exp\left(-\frac{1}{g\nu_F}\right) \tag{1.19}$$

Now we would like to consider finite but small $q$ and $\omega_n = 0$,to calculate the first contribution to GL effective theory(1.11),as we now that there are not time-derivative terms in GL effective theory. Therefore, we should evaluate the integral from (1.17) in this consideration:

$$\chi(0,q) = -\int_p \frac{1 - n_F(\xi_p) - n_F(\xi_{-p+q})}{\xi_p + \xi_{-p+q}}, \tag{1.20}$$

which we rewrites as

$$\begin{aligned}\int_p \frac{1 - n_F(\xi_{p+q/2}) - n_F(\xi_{p-q/2})}{\xi_{p+q/2} + \xi_{p-q/2}} \approx \\ \approx \int_p \frac{1 - 2n_F(\xi_p) + \partial_\xi^2 n_F(\xi_p)(\mathbf{q}\cdot\mathbf{p})/(4m^2)}{2\xi_p} = \\ = \chi(0,0) - \frac{\nu_F \mu q^2}{12m}\int \frac{d\epsilon}{\epsilon}\frac{\partial^2 n_F(\epsilon)}{\partial\epsilon^2} = \chi(0,0) - \frac{\nu_F v_F^2}{24T^2}\frac{7\zeta(3)}{2\pi^2}q^2.\end{aligned} \tag{1.21}$$

In real space, this term gives us $r|\Delta|^2/2 + c|\partial\Delta|^2/2$ contribution to GL effective theory. For static and spatially homogeneous field configurations we have simply have

$$\chi(0,0) = \frac{\nu_F(T - T_c)}{2T_c}|\Delta|^2, \tag{1.22}$$

which corresponds to small deviations near $T_c$. Then let us consider the next term in GL effective theory. Straightforward calculation gives (we explicitly show it in Appendix) that

$$S^{(4)} = \sum_q \frac{1}{2\cdot 2}\,\mathrm{tr}\,(\mathcal{G}_0\Delta)^4 = -\frac{\nu_F}{4T_c^2}\frac{7\zeta(3)}{32\pi^2} \tag{1.23}$$

We see that the key parameter of the expansion is $|\Delta|/T \ll 1$. So, we can truncate other terms in the limit of low temperatures and obtain the following expansion for effective action from (1.8) and (1.9):

$$S_{\rm GL}[\bar{\Delta}, \Delta] = \beta \int d^3 r \left[\frac{r}{2}|\Delta|^2 + \frac{c}{2}|\partial\Delta|^2 + u|\Delta|^4\right], \tag{1.24}$$

which is nothing but the seminal GL effective theory. We would like to emphasize that using field-theoretical approach we obtain the expansion which was initially obtained *phenomenologically.* Moreover, we demonstrate how coefficients of this expansion can be calculated from *microscopic* theory. Finally, let us focus on the *mean-field* approximation which means that $\Delta = \Delta_0 \equiv \text{const}$. Variation of GL-functional (now we neglect gradient term) gives order parameter value which is minimizing the energy functional of superconductor:

$$\left.\frac{\delta S_{\rm GL}[\Delta, \bar{\Delta}]}{\delta\Delta}\right|_{\Delta=\Delta_0} = \bar{\Delta}_0\left(\frac{r}{2} + 2u|\Delta_0|^2\right) = 0 \to \Delta_0 = \sqrt{-\frac{r}{4u}}, \tag{1.25}$$

where we substitute calculated values of the expansion coefficients and find that

$$\Delta_0^2 = \frac{8\pi^2}{7\xi(3)} T_c(T_c - T), \tag{1.26}$$

which is only valid near $T_c$. Singular behavior of order parameter in the limit $T \to T_c$ clearly hints at phase transition. To prove this hint, we calculate the heat capacity near $T_c$. To do it, we substitute (1.26) into GL-expansion and find that free energy difference between superconductor and normal metal near $T_c$ is given by

$$\frac{F_s - F_n}{V} = -\frac{4\pi^2}{7\zeta(3)} \nu_F (T - T_c)^2, \tag{1.27}$$

which clearly that superconducting state has lower energy than normal metal. Then, for entropy difference we have

$$\frac{S_s - S_n}{V} = -\frac{4\pi^2 T_c}{7\zeta(3)}\left(1 - \frac{T}{T_c}\right)^2 \tag{1.28}$$

and we see that superconducting phase also has lower entropy. Finally, for heat capacity one can find

$$\frac{C_s - C_n}{V} = \frac{8\pi^2 \nu_0}{7\zeta(3)}(T - T_c)^2. \tag{1.29}$$

Keeping in mind that $C_n = 2\pi^2 \nu_F T/3$, we find that in $T = T_c$ heat capacity hast discontinuity

$$\frac{C_s - C_n}{C_n} = \frac{12}{\zeta(3)}. \tag{1.30}$$

All obtained results prove that transition from normal phase to superconducting is the second order phase transition. We have seen that all thermodynamic quantities have universal behavior near $T = T_c$. Therefore, the main conclusion of this section is that superconductivity phenomenon is the second order phase transition.

## 1.2 Bogoliubov-deGennes equation

Instead of writing equations of level population and anomalous average, it is possible to consider time depended Bogoliubov transformation.

$$\begin{aligned} b_{q-} =& u_q a_{q,-} + v_q a^\dagger_{-q,+}, \\ b_{q+} =& u_q a_{q,+} - v_q a^\dagger_{-q,-}. \end{aligned}$$

Now assuming that these coefficients satisfy the matrix Schroedinger equation we can write Bogoliubov-deGennes equation:

$$i\partial_T \begin{Bmatrix} u_q \\ v_q \end{Bmatrix} = \begin{Bmatrix} \epsilon_q & \Delta \\ \Delta^* & -\epsilon_q \end{Bmatrix} \times \begin{Bmatrix} u_q \\ v_q \end{Bmatrix}. \tag{1.31}$$

These equations can be modified by adding time-dependent gap equation:

$$\Delta(T) = g \int \frac{d^3q}{(2\pi)^3} u_q(T) v_q^*(T). \tag{1.32}$$

Our notations (2.1) can be write in terms of $u$ and $v$:

$$\kappa_q = u_q v_q^* \; n_q = v_q^2, \; 1 - n_q = u_q^2, \tag{1.33}$$

so:

$$\frac{d}{dT}\kappa_q = \partial_T u_q \times v_q^* + u_q \times \partial_T v_q^*, \tag{1.34}$$

$$\frac{d}{dT} n_q = \partial_T v_q \times v_q^* + v_q \times \partial_T v_q^*. \tag{1.35}$$

Using (1.31) we rewright RHS of the (1.34):

$$\frac{d}{dT}\kappa_q = -i\Delta(v_q^2 - u_q^2) - 2i\epsilon_q v_q^* u_q = -ig \int \frac{d^3p}{(2\pi)^3} \kappa_p (2n_q - 1) - 2i\epsilon_q \kappa_q, \tag{1.36}$$

$$\frac{d}{dT} n_q = -i\Delta^* u_q v_q^* + i\Delta u_q^* v_q = -ig \int \frac{d^3p}{(2\pi)^3} [k_q k_p^* - k_q^* k_p], \tag{1.37}$$

which are equivalent to (2.7).

# 2 First order

We consider BCS theory of fermionic gas with attractive interaction. We calculate the time evolution of “normal and anomalous“ expectation values of the creation and annihilation operators: $\langle a^+ a\rangle$ and $\langle aa\rangle$, respectively. We use Heisenberg's equations and assume for simplicity the invariance of the initial state under spatial translations. The extention to slightly spatially inhomogeneous situations is straightforward. Our notations are:

$$(N_{pq})_{\alpha\beta} = \left\langle a^\dagger_{p\alpha} a_{q\beta}\right\rangle = \delta(p-q)\begin{Bmatrix} n_p & 0 \\ 0 & n_p \end{Bmatrix},\ (X_{pq})_{\alpha\beta} = \langle a_{p\alpha} a_{q\beta}\rangle = \delta(p+q)\begin{Bmatrix} 0 & \chi_p \\ -\chi_p & 0 \end{Bmatrix}. \tag{2.1}$$

Where $a_{p\alpha}$ and $a^\dagger_{p\alpha}$ are the annihilation and creation operators. The free Hamiltonian of the theory under consideration is as follows:

$$H_0 = \sum_\alpha \int \frac{d^3p}{(2\pi)^3} \epsilon_p a^\dagger_{p\alpha} a_{p\alpha}, \quad \epsilon_p = \frac{\vec{p}^{\,2}}{2m} - \mu. \tag{2.2}$$

The interaction term in the interaction representation is:

$$H_{int}(t) = -g \int \frac{d^3p_1..d^3p_4}{(2\pi)^{12}} \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) a^\dagger_{p_1+} a^\dagger_{p_2-} a_{p_3-} a_{p_4+} e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})t}. \tag{2.3}$$

The first order Heisenberg's equations for the level population and anomalous average are:

$$\frac{d}{dT}\left\langle a^\dagger_{q_1+} a_{q_2+}\right\rangle = i\left\langle [H_{int}(T), a^\dagger_{q_1+} a_{q_2+}]\right\rangle, \tag{2.4}$$

$$\frac{d}{dT}\langle a_{q_1+} a_{q_2-}\rangle = i\langle [H_{int}(T), a_{q_1+} a_{q_2-}]\rangle. \tag{2.5}$$

Using Wick's theorem on the RHS of (2.4)(with “anomalous“ expectation values included) we get the following system of equations:

$$\frac{d}{dT} n_q = ig\frac{d^3p}{(2\pi)^3}\Big[e^{i(2\epsilon_q - 2\epsilon_p)T}\chi^*_q\chi_p - e^{i(2\epsilon_p - 2\epsilon_q)T}\chi_q\chi^*_p\Big], \tag{2.6}$$

$$\frac{d}{dT}\chi_q = -ig\frac{d^3p}{(2\pi)^3}\Big[e^{i(2\epsilon_q - 2\epsilon_p)T}\Big[-(1-n_q)\chi_p + n_q\chi_p\Big] - 2n_p\chi_q\Big],$$

where we have used the notations (2.1). Pay attantion that as follows from (2.7) $\chi_p$ remains zero, if it was initially zero. That is because the Hamiltonian (2.2), (2.3) commutes with the total number of particles: $N = \sum_{p,\alpha} a^{+}_{p,\alpha} a_{p,\alpha}$. It will be shown below that this is reasonable to rewrite equations in the following form:

$$\frac{d}{dT} n_q = ig \frac{d^3p}{(2\pi)^3} \Big[ \kappa_q^* \kappa_p - \kappa_q \kappa_p^* \Big], \tag{2.7}$$

$$\frac{d}{dT} \kappa_q + 2i\epsilon_q \kappa_q = ig \frac{d^3p}{(2\pi)^3} \Big[ (1 - 2n_q) \kappa_p \Big].$$

# 3 Dynamical Gorkov's equations

We use equation of motion for $\psi$ to write equations for Green's functions:

$$i\partial_t\psi_+ = -\Big(\frac{\nabla^2}{2m} + \mu\Big)\psi_+ - g\psi_-^+\psi_-\psi_+, \tag{3.1}$$

$$i\partial_t\psi_- = -\Big(\frac{\nabla^2}{2m} + \mu\Big)\psi_- + g\psi_+^+\psi_-\psi_+. \tag{3.2}$$

We use these equations to derive Dyson - Schwinger equations for the following functions:

$$iG(t_1,t_2,x) = \big\langle T\psi_+(t_1,x_1)\psi_+^+(t_2,x_2)\big\rangle\,, \tag{3.3}$$

$$iF(t_1,t_2,x) = \langle T\psi_+(t_1,x_1)\psi_-(t_2,x_2)\rangle\,, \tag{3.4}$$

$$i\tilde{G}(t_1,t_2,x) = \big\langle T\psi_+^+(t_1,x_1)\psi_+(t_2,x_2)\big\rangle\,, \tag{3.5}$$

where $x = x_2 - x_1$. We consider states that are invariant under spatial translations. Denoting $T = \frac{t_1+t_2}{2}$ and $\tau = t_2 - t_1$, we obtain:

$$\partial_T = \partial_{t_1} + \partial_{t_2}, \quad \partial_\tau = \frac{1}{2}(\partial_{t_2} - \partial_{t_1}). \tag{3.6}$$

Using notation:

$$\Xi(T) = iF(T,T,0), \tag{3.7}$$

we obtain the following equations after the Fourier transformation in spatial directions:

$$i\partial_T G(t_1,t_2,q) = -g\Big[F(t_1,t_2,q)\Xi^*(t_2) + F^+(t_1,t_2,q)\Xi(t_1)\Big], \tag{3.8}$$

$$i\partial_T F(t_1,t_2,q) = +2\epsilon_q F(t_1,t_2,q) + g\Big[\tilde{G}(t_1,t_2,q)\Xi(t_1) - G(t_1,t_2,x)\Xi(t_2)\Big], \tag{3.9}$$

$$i\partial_T F^+(t_1,t_2,q) = -2\epsilon_q F^+(t_1,t_2,q) + g\Big[\tilde{G}(t_1,t_2,q)\Xi^*(t_2) - G(t_1,t_2,q)\Xi^* t_1)\Big], \tag{3.10}$$

$$i\partial_T \tilde{G}(t_1,t_2,q) = g\Big[F((t_1,t_2,q)\Xi^*(t_1) + F^+(t_1,t_2,q)\Xi(t_2)\Big], \tag{3.11}$$

$$i\partial_\tau G(t_1,t_2,q) = -\epsilon_q G(t_1,t_2,q) - \frac{g}{2}\Big[F(t_1,t_2,q)\Xi^*(t_2) - F^+(t_1,t_2,q)\Xi(t_1)\Big] + \delta(t_1-t_2), \tag{3.12}$$

$$i\partial_\tau F(t_1,t_2,q) = -\frac{g}{2}\Big[\tilde{G}(t_1,t_2,q)\Xi(t_1) + G(t_1,t_2,x)\Xi(t_2)\Big], \tag{3.13}$$

$$i\partial_\tau F^+(t_1,t_2,q) = \frac{g}{2}\Big[\tilde{G}(t_1,t_2,q)\Xi^*(t_2) + G(t_1,t_2,q)\Xi^*(t_1)\Big], \tag{3.14}$$

$$i\partial_\tau \tilde{G}(t_1,t_2,q) = \epsilon_q \tilde{G}(t_1,t_2,q) - \frac{g}{2}\Big[F(t_1,t_2,q)\Xi^*(t_1) - F^+(t_1,t_2,q)\Xi(t_2)\Big] - \delta(t_1-t_2). \tag{3.15}$$

The unit on the RHS (3.12) and (3.15) come from the leap of these functions due to the T-ordering. Assuming that the function $\Xi$ is slow:

$$\Xi(t_1) \approx \Xi(t_2) \approx \Xi(T), \tag{3.16}$$

we make Fourier transformation in $\tau, = t_2 - t_1$ (i.e. fully completing the Wigner transformation):

$$i\partial_T G(T,\omega,q) = -g\Big[F(T,\omega,q)\Xi^*(T) + F^+(T,\omega,q)\Xi(T)\Big], \tag{3.17}$$

$$i\partial_T F(T,\omega,q) = +2\epsilon_q F(T,\omega,q) + g\Big[\tilde{G}(T,\omega,q)\Xi(T) - G(t_1,t_2,x)\Xi(T)\Big], \tag{3.18}$$

$$i\partial_T F^+(T,\omega,q) = -2\epsilon_q F^+(T,\omega,q) + g\Big[\tilde{G}(T,\omega,q)\Xi^*(T) - G(T,\omega,q)\Xi^*(T)\Big], \tag{3.19}$$

$$i\partial_T \tilde{G}(T,\omega,q) = g\Big[F(T,\omega,q)\Xi^*(T) + F^+(T,\omega,q)\Xi(T)\Big], \tag{3.20}$$

$$\omega G(T,\omega,q) = -\epsilon_q G(T,\omega,q) - \frac{g}{2}\Big[F(T,\omega,q)\Xi^*(T) - F^+(T,\omega,q)\Xi(T)\Big] + 1, \quad (3.21)$$

$$\omega F(T,\omega,q) = -\frac{g}{2}\Big[\tilde{G}(T,\omega,q)\Xi(T) + G(T,\omega,q)\Xi(T)\Big], \quad (3.22)$$

$$\omega F^+(T,\omega,q) = \frac{g}{2}\Big[\tilde{G}(T,\omega,q)\Xi^*(T) + G(T,\omega,q)\Xi^*(T)\Big], \quad (3.23)$$

$$\omega \tilde{G}(T,\omega,q) = \epsilon_q \tilde{G}(T,\omega,q) - \frac{g}{2}\Big[F(T,\omega,q)\Xi^*(T) - F^+(T,\omega,q)\Xi(T)\Big] - 1. \quad (3.24)$$

$$(3.25)$$

Now we show that these equations are equivalent to (2.7). From (3.7) and (4.7) we find that:

$$\Xi(T) = iF(T,T,0) = i\int \frac{d\omega d^3p}{(2\pi)^4} F(T,\omega,p) = \int \frac{d^3p}{(2\pi)^3}\kappa_p. \quad (3.26)$$

Now taking the integral over $\omega$ in (3.24), closing the contour in the upper half plane, we get:

$$i\partial_T \int \frac{d\omega}{2\pi}\tilde{G}(T,\omega,q) = g\Big[\int \frac{d\omega}{2\pi}F(T,\omega,q)\Xi^*(T) + \int \frac{d\omega}{2\pi}F^+(T,\omega,q)\Xi(T)\Big], \quad (3.27)$$

$$\text{i.e. } \partial_T n_q = ig\int \frac{d^3p}{(2\pi)^3}\Big[\kappa_q^*\kappa_p - \kappa_q\kappa_p^*\Big], \quad (3.28)$$

where:

$$-\kappa_q^*(T) = i\int \frac{d\omega}{2\pi}F^+(T,\omega,q). \quad (3.29)$$

Similarly from (3.22) we obtain:

$$(\partial_T + 2i\epsilon_q)\kappa_q = ig\int \frac{d^3p}{(2\pi)^3}\Big[(1-2n_q)\kappa_p\Big]. \quad (3.30)$$

Which coincides with (2.7).

## 4 Stationary solution

Let us see that the stationary soultion of (2.7) is equivalent to the solution of the Gorkov's equations. Gorkov's equations are the first order stationary Dyson equation for ordinary and anomalous Green's functions as follows:

$$iG(x_1,t_1,x_2,t_2) = \left\langle T\psi_+(x_1,t_1)\psi_+^+(x_2,t_2)\right\rangle, \tag{4.1}$$

$$iF(x_1,t_1,x_2,t_2) = \left\langle T\psi_+(x_1,t_1)\psi_-(x_2,t_2)\right\rangle. \tag{4.2}$$

Gorkov's equations in spatial and time translation invariant situation($x = x_2 - x_1,\ \tau = t_2 - t_1$) are as follows:

$$(\omega - \epsilon_p)\, G(\omega,p) + gF(x=0,\tau=0)F^+(\omega,p) + gG(\omega,p)G(x=0,\tau=0) = 1,$$

$$(\omega + \epsilon_p)\, F^+(\omega,p) + gF^+(x=0,\tau=0)G(\omega,p) + gF(\omega,p)G(x=0,\tau=0) = 0.$$

Absorbing terms with $G(x=0,\tau=0)$ into the renormalization of the chemical potential , we get the following solutions of the equations after Fourier transform:

$$G(\omega,p) = \frac{\omega+\epsilon_p}{(\omega - E_p + i0)(\omega + E_p - io)}, \quad F(\omega,p) = -\frac{g\Xi}{(\omega - E_p - i0)(\omega + E_p + io)}, \tag{4.3}$$

where $E_p = \sqrt{\Delta^2 + \epsilon_p^2}$ and

$$\Delta = igF(0) = ig\int \frac{d\omega d^3p}{(2\pi)^4} F(\omega,p). \tag{4.4}$$

Now let us see that the same solutions follows from (2.7). First there is the following relation between anomalous Green's function and the anomalous expectation value:

$$F(0) = F(x,t|x,t) = \int\int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\left\langle a_{p+}a_{q-}\right\rangle e^{ipx+iqx}e^{-i\epsilon_p t - i\epsilon_q t} = \int \frac{d^3p}{(2\pi)^3}\chi_p e^{-2i\epsilon_p t}. \tag{4.5}$$

Hence, to obtain the spatially homogeneous situation we have to represent:

$$\chi_p = \kappa_p e^{2i\epsilon_p t}, \tag{4.6}$$

and in stationary situation we have that:

$$\frac{d}{dt}\kappa_p = 0,$$

as $\Delta$ is a constant in equilibrium. Hence from (4.3) we get that:

$$\kappa_p = i\int \frac{d\omega}{2\pi} F(\omega, p) = \frac{\Delta}{2E_p}, \quad -n_p = i\int \frac{d\omega}{2\pi} G(\omega, p) = \frac{E_p - \epsilon_p}{2E_p}. \tag{4.7}$$

In such a case equations (2.7) transform into:

$$\frac{d}{dT} n_q = 0 \quad \text{and} \quad 2\epsilon_q \kappa_q = \frac{g}{V}\sum_p \Big[(1 - 2n_q)\kappa_p + 2n_p\kappa_q\Big]. \tag{4.8}$$

Reason why we exclude second term in $\kappa$ equation is that it is possible to absorb the second term on the RHS of the second equation into the chemical potential renormalization. Then, from the second relation in (4.8) we get the gap equation:

$$\frac{g}{V}\sum_p (1 - 2n_q)\kappa_p = g\int \frac{d^3p}{(2\pi)^3}\frac{\epsilon_q}{E_q}\frac{\Delta}{2E_p} = 2\epsilon_q\kappa_q \frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{\Delta^2 + \epsilon_p^2}}. \tag{4.9}$$

Hence, from (4.8) it follows, that:

$$\frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{\Delta^2 + \epsilon_p^2}} = 1. \tag{4.10}$$

# 5 Kinetic equations for $n_p$ and $\kappa_p$ at $g^2$ order

We also can derive equations of the second order in $g$, which can be related to the resummation of the "sunset" diagrams. The Heisenberg's equations for the level population and for the anomalous average are as follows:

$$\frac{d}{dT}\left\langle a^\dagger_{q_1+} a_{q_2+}\right\rangle = i\left\langle\left[H_{int}(T), a^\dagger_{q_1+} a_{q_2+}\right]\right\rangle = -ig\int\frac{d^3p_1..d^3p_4}{(2\pi)^{12}}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\times \tag{5.1}$$

$$\times e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})T}\left[\delta(\vec{q}_1-\vec{p}_4)\left\langle a^\dagger_{p_1+}a^\dagger_{p_2-}a_{p_3-}a_{q_2+}\right\rangle - \delta(\vec{p}_1-\vec{q}_2)\left\langle a^\dagger_{q_1+}a^\dagger_{p_2-}a_{p_3-}a_{p_4+}\right\rangle\right],$$

and

$$\frac{d}{dT}\left\langle a_{q_1+} a_{q_2-}\right\rangle = i\left\langle\left[H_{int}(T), a_{q_1+} a_{q_2-}\right]\right\rangle = -ig\int\frac{d^3p_1..d^3p_4}{(2\pi)^{12}}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\times \tag{5.2}$$

$$\times e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})T}\left[\delta(\vec{p}_2-\vec{q}_2)\left\langle a_{q_1+}a^\dagger_{p_1+}a_{p_3-}a_{p_4+}\right\rangle - \delta(\vec{q}_1-\vec{p}_1)\left\langle a^\dagger_{p_2-}a_{p_3-}a_{p_4+}a_{q_2-}\right\rangle\right].$$

To find the second order equations for the level population and for the anomalous average we have to derive the equations for the expectation values appearing on the right hand sides of (5.1) and (5.2). Details of calculations are in Appendix. E.g.

$$\frac{d}{dT'}\left\langle a^\dagger_{p_1+}a^\dagger_{p_2-}a_{p_3-}a_{q_2+}\right\rangle = i\left\langle [H_{int}(T'), a^\dagger_{p_1+}a^\dagger_{p_2-}a_{p_3-}a_{q_2+}]\right\rangle.$$

Pay attention that right hand side of this equation depends only on $T'$ rather than on $T$. Meanwhile the right hand sides of (5.1) and (5.2) are the functions of T. Then only after the extraction of the fast oscillating part from $\chi$ according to (4.6), we can use the kinetic approximation and take the integral over $T'$.
Namely, we assume that while approaching the ground state (Cooper pair condensate) $\chi$ oscillates the same way as in (4.6):

$$\chi_p(T') = \kappa_p(T')e^{2i\epsilon_p T'},$$

where $\kappa(T')$ is a slow function of $T'$ in the kinetic approximation.

Using Wick's contractions, we find the following equations:

$$\frac{d}{dT}n_q = -g^2\int_{T_0}^{T} dT^{'}\Bigg[\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\Bigg\{\delta^3(\vec{k}+\vec{p}-\vec{l}-\vec{q})\cos\Big((\epsilon_p+\epsilon_k-\epsilon_q-\epsilon_l)(T-T^{'})\Big)\times$$

$$\times\Big[n_q n_l(1-n_p)(1-n_k)-(1-n_q)(1-n_l)n_p n_k\Big]-$$

$$-\delta^3(\vec{p}+\vec{k}-\vec{l}+\vec{q})\Big[e^{i(\epsilon_p-\epsilon_q)(T+T^{'})}e^{i(\epsilon_l-\epsilon_k)(T-T^{'})}\Big[\chi_p^* n_k(1-n_l)\chi_q-\chi_p^*(1-n_k)n_l\chi_q\Big]+h.c.\Big]-$$

$$-\delta^3(\vec{p}+\vec{k}+\vec{l}-\vec{q})\Big[e^{i(\epsilon_p-\epsilon_l)(T+T^{'})}e^{i(\epsilon_k-\epsilon_q)(T-T^{'})}\Big[n_q\chi_p^*\chi_l(1-n_k)-(1-n_q)\chi_p^*\chi_l n_k\Big]+h.c.\Big]\Bigg\}+$$

$$\int\frac{d^3pd^3k}{(2\pi)^6}\Bigg\{-\Big[e^{i(2\epsilon_p-2\epsilon_q)T}e^{i(2\epsilon_q-2\epsilon_k)T^{'}}\Big[-n_q n_q\chi_k\chi_p^*+(1-n_q)(1-n_q)\chi_k\chi_p^*\Big]+h.c.\Big]-$$

$$-\Big[e^{i(2\epsilon_p-2\epsilon_q)T}e^{i(2\epsilon_k-2\epsilon_p)T^{'}}\Big[\chi_k^*(1-n_p)(1-n_p)\chi_q-\chi_k^* n_p n_p\chi_q\Big]+h.c.\Big]\Bigg\}\Bigg],$$

$$\frac{d}{dT}\chi_q = -g^2\int_{T_0}^{T} dT^{'}\Bigg[$$

$$\int\frac{d^3pd^3k}{(2\pi)^6}\Bigg\{4\Big[n_q n_k\chi_q n_p+(1-n_q)n_k\chi_q n_p\Big]-$$

$$-2e^{i(2\epsilon_q-2\epsilon_p)T}e^{i(2\epsilon_q-2\epsilon_k)T^{'}}\Big[n_q\chi_q^*\chi_p\chi_k+(1-n_q)\chi_q^*\chi_p\chi_k\Big]+$$

$$+2e^{i(2\epsilon_q-2\epsilon_k)T^{'}}\Big[-n_q n_q\chi_k n_p+(1-n_q)(1-n_q)\chi_k n_p\Big]+$$

$$+2e^{i(2\epsilon_q-2\epsilon_k)T}\Big[n_k n_p(1-n_q)\chi_k+(1-n_k)n_p(1-n_q)\chi_k\Big]-$$

$$-2e^{i(2\epsilon_q-2\epsilon_p)T}e^{i(2\epsilon_k-2\epsilon_q)T^{'}}\Big[-\chi_k^*\chi_q(1-n_q)\chi_p-\chi_k^*\chi_q n_q\chi_p\Big]+$$

$$+2e^{i(2\epsilon_q-2\epsilon_k)T}\Big[n_k n_p n_q\chi_k+(1-n_k)n_p n_q\chi_k\Big]+$$

$$+e^{i(2\epsilon_q-2\epsilon_k)T}e^{i(2\epsilon_k-2\epsilon_p)T^{'}}\Big[-n_k n_k(1-n_q)\chi_p+(1-n_k)(1-n_k)(1-n_q)\chi_p-$$

$$-n_k n_k n_q\chi_p+(1-n_k)(1-n_k)n_q\chi_p\Big]\Bigg\}+$$

$$\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\Bigg\{-2\delta^3(\vec{q}-\vec{p}+\vec{l}+\vec{k})e^{i(\epsilon_q+\epsilon_p-\epsilon_k-\epsilon_l)(T+T^{'})}\Big[-n_q\chi_p^*\chi_k\chi_l-(1-n_q)\chi_p^*\chi_k\chi_l\Big]+$$

$$+2\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})e^{i(\epsilon_l-\epsilon_k)(T-T^{'})}e^{i(\epsilon_q-\epsilon_p)(T+T^{'})}\Big[-n_q n_k(1-n_l)\chi_p-(1-n_q)(1-n_k)n_l\chi_p\Big]-$$

$$-2\delta^3(\vec{q}-\vec{k}-\vec{l}-\vec{p})e^{i(\epsilon_k-\epsilon_p)(T+T^{'})}e^{i(\epsilon_q-\epsilon_l)(T-T^{'})}\Big[\chi_k^* n_l\chi_q\chi_p+\chi_k^*(1-n_l)\chi_q\chi_p\Big]\Bigg\}\Bigg].$$

And also there is the complex conjugate equations for $\chi_q^*$.

Extracting the fast oscillationg part from $\chi_p$ in accordance with(4.6) and assuming that $\kappa_p$ and $n_p$ are slow functions of $T$ and, then, taking the integral over $T'$ in the limits $T_0 \to -\infty$ and $T \to +\infty$ we obtain the following system of kinetic equations:

$$\frac{d}{dT}n_q = -g^2$$

$$\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\Big\{\delta^3(\vec{k}+\vec{p}-\vec{l}-\vec{q})\delta(\epsilon_p+\epsilon_k-\epsilon_q-\epsilon_l)\Big[n_qn_l(1-n_p)(1-n_k)-(1-n_q)(1-n_l)n_pn_k\Big]-$$

$$-\delta^3(\vec{p}+\vec{k}-\vec{l}+\vec{q})\Big[\delta(\epsilon_p+\epsilon_l-\epsilon_q-\epsilon_k)[\kappa_p^*n_k(1-n_l)\kappa_q-\kappa_p^*(1-n_k)n_l\kappa_q]+h.c.\Big]-$$

$$-\delta^3(\vec{p}+\vec{k}+\vec{l}-\vec{q})\Big[\delta(\epsilon_p+\epsilon_k-\epsilon_l-\epsilon_q)[n_q\kappa_p^*\kappa_l(1-n_k)-(1-n_q)\kappa_p^*\kappa_ln_k]+h.c.\Big]\Big\}+$$

$$\int \frac{d^3pd^3k}{(2\pi)^6}\Big\{-\Big[[\delta(\epsilon_p-\epsilon_q)\Big[-n_qn_q\kappa_k\kappa_p^*+(1-n_q)(1-n_q)\kappa_k\kappa_p^*\Big]+h.c.\Big]-$$

$$-\delta(\epsilon_p-\epsilon_q)\Big[\Big[\kappa_k^*(1-n_p)(1-n_p)\kappa_q-\kappa_k^*n_pn_p\kappa_q\Big]+h.c.\Big]\Big\},$$

and

$$\left(\frac{d}{dT}+2i\epsilon_q\right)\kappa_q = -g^2$$

$$\int \frac{d^3pd^3k}{(2\pi)^6}\Big\{4e^{-2i\epsilon_qt}\delta(\epsilon_q)\Big[n_qn_k\kappa_qn_p+(1-n_q)n_k\kappa_qn_p\Big]-$$

$$-2\delta(\epsilon_p)\Big[n_q\kappa_q^*\kappa_p\kappa_k+(1-n_q)\kappa_q^*\kappa_p\kappa_k\Big]+$$

$$+2e^{-i(2\epsilon_q)t}\delta(\epsilon_q)\Big[-n_qn_q\kappa_kn_p+(1-n_q)(1-n_q)\kappa_kn_p\Big]+$$

$$+2\delta(\epsilon_k)\Big[n_kn_p(1-n_q)\kappa_k+(1-n_k)n_p(1-n_q)\kappa_k\Big]-$$

$$-2\delta(\epsilon_p)\Big[-\kappa_k^*\kappa_q(1-n_q)\kappa_p-\kappa_k^*\kappa_qn_q\kappa_p\Big]+$$

$$+2\delta(\epsilon_k)\Big[n_kn_pn_q\kappa_k+(1-n_k)n_pn_q\kappa_k\Big]+$$

$$+\delta(\epsilon_k)\Big[-n_kn_k(1-n_q)\kappa_p+(1-n_k)(1-n_k)(1-n_q)\kappa_p-n_kn_kn_q\kappa_p+(1-n_k)(1-n_k)n_q\kappa_p\Big]\Big\}+$$

$$\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\Big\{-2\delta^3(\vec{q}-\vec{p}+\vec{l}+\vec{k})\delta(\epsilon_q+\epsilon_l+\epsilon_k-\epsilon_p)\Big[-n_q\kappa_p^*\kappa_k\kappa_l-(1-n_q)\kappa_p^*\kappa_k\kappa_l\Big]+$$

$$+2\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})\delta(\epsilon_q+\epsilon_p+\epsilon_k-\epsilon_l)\Big[-n_qn_k(1-n_l)\kappa_p-(1-n_q)(1-n_k)n_l\kappa_p\Big]-$$

$$-2\delta^3(\vec{q}-\vec{k}-\vec{l}-\vec{p})\delta(\epsilon_q+\epsilon_l+\epsilon_p-\epsilon_k)\Big[\kappa_k^*n_l\kappa_q\kappa_p+\kappa_k^*(1-n_l)\kappa_q\kappa_p\Big]\Big\}.$$

## 5.1 Interpetation of equation for level population

Let us give the physical interpretation of various contribution to the collision integral in the equations for $n_q$. The standard part of the collision integral is as follows:

$$-\delta^3(\vec{k}+\vec{p}-\vec{l}-\vec{q})\delta(\epsilon_p+\epsilon_k-\epsilon_q-\epsilon_l)\Big[n_q n_l(1-n_p)(1-n_k)-(1-n_q)(1-n_l)n_p n_k\Big].$$

This term looks like particle with momentum q changes the momentum to k due to interacting with the state. But we should admit that sign of this term is opposite:

$$\delta^3(\vec{p}+\vec{k}+\vec{l}-\vec{q})\Bigg[\delta(\epsilon_p+\epsilon_k-\epsilon_l-\epsilon_q)[n_q\kappa_p^*\kappa_l(1-n_k)-(1-n_q)\kappa_p^*\kappa_l n_k]+h.c.\Bigg].$$

This term looks like particle with momentum k changes the momentum to l due to interacting with the state.

$$\delta^3(\vec{p}+\vec{k}-\vec{l}+\vec{q})\Bigg[\delta(\epsilon_p+\epsilon_l-\epsilon_q-\epsilon_k)[\kappa_p^* n_k(1-n_l)\kappa_q-\kappa_p^*(1-n_k)n_l\kappa_q]+h.c.\Bigg].$$

These two terms correspond to the second order bubble diagrams, which we do not consider in kinetic equation:

$$-\Bigg[[\delta(\epsilon_p-\epsilon_q)\Big[-n_q n_q\kappa_k\kappa_p^*+(1-n_q)(1-n_q)\kappa_k\kappa_p^*\Big]+h.c.\Bigg].$$

$$-\delta(\epsilon_p-\epsilon_q)\Bigg[\Big[\kappa_k^*(1-n_p)(1-n_p)\kappa_q-\kappa_k^* n_p n_p\kappa_q\Big]+h.c.\Bigg].$$

## 5.2 Interpretation of equation for $\kappa$

Let us give the physical interpretation of various contribution to the collision integral in the equations for $\kappa_q$. Terms are canceled due to fast oscillation :

$$4e^{-2i\epsilon_q T}\delta(\epsilon_q)\Big[n_q n_k \kappa_q n_p + (1-n_q)n_k\kappa_q n_p\Big],$$

and

$$2e^{-i(2\epsilon_q)T}\delta(\epsilon_q)\Big[-n_q n_q \kappa_k n_p + (1-n_q)(1-n_q)\kappa_k n_p\Big].$$

Two terms above and five terms below correspond to the second order bubble diagrams, which we do not consider in kinetic equation:

$$-2\delta(\epsilon_p)\Big[n_q\kappa_q^*\kappa_p\kappa_k + (1-n_q)\kappa_q^*\kappa_p\kappa_k\Big],$$

and

$$-2\delta(\epsilon_p)\Big[-\kappa_k^*\kappa_q(1-n_q)\kappa_p - \kappa_k^*\kappa_q n_q\kappa_p\Big],$$

and

$$\delta(\epsilon_k)\Big[-n_k n_k(1-n_q)\kappa_p + (1-n_k)(1-n_k)(1-n_q)\kappa_p - n_k n_k n_q\kappa_p + (1-n_k)(1-n_k)n_q\kappa_p\Big],$$

and

$$2\delta(\epsilon_k)\Big[n_k n_p(1-n_q)\kappa_k + (1-n_k)n_p(1-n_q)\kappa_k\Big],$$

and

$$2\delta(\epsilon_k)\Big[n_k n_p n_q\kappa_k + (1-n_k)n_p n_q\kappa_k\Big].$$

Terms are semi-canceled due to energy-momentum conservation. As it impossible for particle to annihilate into three particles, but there are situations when these delta-functions are non-zero. :

$$-2\delta^3(\vec{q}-\vec{p}+\vec{l}+\vec{k})\delta(\epsilon_q+\epsilon_l+\epsilon_k-\epsilon_p)\Big[-n_q\kappa_p^*\kappa_k\kappa_l - (1-n_q)\kappa_p^*\kappa_k\kappa_l\Big],$$

and

$$2\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})\delta(\epsilon_q+\epsilon_p+\epsilon_k-\epsilon_l)\Big[-n_q n_k(1-n_l)\kappa_p - (1-n_q)(1-n_k)n_l\kappa_p\Big],$$

and

$$-2\delta^3(\vec{q}-\vec{k}-\vec{l}-\vec{p})\delta(\epsilon_q+\epsilon_l+\epsilon_p-\epsilon_k)\Big[\kappa_k^* n_l\kappa_q\kappa_p + \kappa_k^*(1-n_l)\kappa_q\kappa_p\Big].$$

For example, when $\epsilon_q \neq 0$ first terms transform to :

$$-2\delta^3(\vec{q}-\vec{p})\delta(\epsilon_q-\epsilon_p)\delta(\epsilon_k)\Big[-n_q\kappa_p^*\kappa_k\kappa_k - (1-n_q)\kappa_p^*\kappa_k\kappa_k\Big].$$

## 5.3 Complete equations

Excluding terms corresponding to the bubble diagrams and leaving terms corresponding to the sunset diagrams we obtain complete kinetic equations for $n_q$ and $\kappa_q$:

$$\frac{d}{dT}n_q = g^2\Bigg[\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\Bigg\{\delta^3(\vec{k}+\vec{l}-\vec{p}-\vec{q})\delta(\epsilon_k+\epsilon_l-\epsilon_q-\epsilon_p)\Big[(1-n_q)(1-n_p)n_ln_k-n_qn_p(1-n_l)(1-n_k)\Big]-$$
$$+\delta^3(\vec{k}+\vec{l}-\vec{q}-\vec{p})\Big[\delta(\epsilon_k+\epsilon_l-\epsilon_q-\epsilon_p)[\kappa_k^*n_p(1-n_l)\kappa_q-\kappa_k^*(1-n_p)n_l\kappa_q]+h.c.\Big]+$$
$$+\delta^3(\vec{k}+\vec{l}-\vec{q}-\vec{p})\Big[\delta(\epsilon_k+\epsilon_l-\epsilon_p-\epsilon_q)[n_q\kappa_l^*\kappa_p(1-n_k)-(1-n_q)\kappa_l^*\kappa_pn_k]+h.c.\Big]\Bigg\}\Bigg]. \quad (5.3)$$

$$\left(\frac{d}{dT}+2i\epsilon_q\right)\kappa_q = g^2\Bigg[\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\Bigg\{2\delta^3(\vec{q}+\vec{p}+\vec{k}-\vec{l})\delta(\epsilon_q+\epsilon_p+\epsilon_k-\epsilon_l)\Big[-n_q\kappa_l^*\kappa_k\kappa_p-(1-n_q)\kappa_l^*\kappa_k\kappa_p\Big]-$$
$$-2\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})\delta(\epsilon_q+\epsilon_p+\epsilon_k-\epsilon_l)\Big[-n_qn_k(1-n_l)\kappa_p-(1-n_q)(1-n_k)n_l\kappa_p\Big]+$$
$$+2\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})\delta(\epsilon_q+\epsilon_k+\epsilon_p-\epsilon_l)\Big[\kappa_l^*n_k\kappa_q\kappa_p+\kappa_l^*(1-n_k)\kappa_q\kappa_p\Big]\Bigg\}\Bigg]. \quad (5.4)$$

# 6 Conclusion and Acknowledgements

There are known solutions for linear oscillating deviations of order parameter $\Delta(T)$ in the first order in $g$, we are interested in description these solutions in terms of our equations (2.6). As our equations appeared to be more complicated than we expected, it is not obvious how to rewrite known equations for oscillations around the static solution (4.7).

The main result of our work are kinetic equations (5.3) and (5.4) in the second order in $g$. Our further task is to find solutions of these equations, we expect that there will be solutions that describe some evolution from non-vacuum order parameter to the right order parameter (4.7).

The main feature of our work is that our method allow to consider more complicated interactions like e.g. $\int \frac{dp_1^3..dp_4^3}{(2pi)^4}\psi^+_{\vec{p}_1}\psi^+_{\vec{p}_2}\psi_{\vec{p}_3}\psi_{\vec{p}_4}A_{\vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_4}$ with non spatial pairing $\langle a_{\vec{p}}a_{\vec{q}}\rangle \neq \delta^3(\vec{p}-\vec{q})\chi_p$, which can hopefully be key to understanding high-temperature superconductivity or condensation in $^3He$.

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# A Coefficients of GL expansion

## A.1 2-nd power coefficient

For the second coefficient, we use expansion near $T_c$ for distribution function:

$$n_F(\epsilon, T) = n_F(\epsilon, T_c) - \frac{\epsilon(T - T_c)}{T}\frac{\partial n_F}{\partial \epsilon} \tag{A.1}$$

and substitute this expansion into (1.18). It gives

$$S^{(2)} = \frac{\nu_F}{2}\frac{T - T_c}{T_c}|\Delta|^2 \tag{A.2}$$

## A.2 4-th power coefficient

$$S^{(4)} = \sum_q \frac{1}{2 \cdot 2} \operatorname{tr}(\mathcal{G}_0\Delta)^4 = \frac{T|\Delta|^4}{4}\sum_n \int_p \frac{1}{\left(\omega_n^2 + \xi_p^2\right)^2}, \tag{A.3}$$

where we have used that $\xi_p = \xi_{-p}$ and connection between particle and hole Green functions. Using (**??**), one can easily obtain

$$\sum_n \frac{1}{((2n+1)^2\pi^2 + (\beta\xi_p)^2)^2} = \frac{1}{4(\beta\xi_p)^3}\tanh\frac{\beta\xi_p}{2} - \frac{1}{8(\beta\xi_p)^2}\frac{1}{\cosh^2(\beta\xi_p/2)} \tag{A.4}$$

Introducing new variable $\beta\xi_p/2 = x$, we find that

$$\sum_n \frac{1}{((2n+1)^2\pi^2 + (\beta\xi_p)^2)^2} \to \frac{1}{32x^2}\left(\frac{\tanh x}{x} - \frac{1}{\cosh^2 x}\right) \tag{A.5}$$

Then we calculate integral over momenta by approximating integration over Fermi surface and case of low temperatures, which gives

$$\int_0^\infty \frac{dx}{x^2}\left(\frac{\tanh x}{x} - \frac{1}{\cosh^2 x}\right) = -\frac{7\zeta(3)}{\pi^2}, \tag{A.6}$$

where we have used Weirstrass theorem, which allows us to represent $\cosh x$ as infinite product

$$\cosh x = \prod_{n \geq 0}\left(1 + \frac{4x^2}{(2n+1)^2\pi^2}\right) \tag{A.7}$$

and then apply logarithmic derivative to obtain

$$\frac{\tanh x}{x} = 8 \sum_{n \geq 0} \frac{1}{(2n+1)^2 + 4x^2}, \tag{A.8}$$

which we differentiate this one more time,

$$\frac{1}{\cosh^2 x} = 8 \sum_{n \geq 0} \frac{(2n+1)^2 \pi^2 - 4x^2}{((2n+1)^2 \pi^2 + 4x^2)^2}, \tag{A.9}$$

then calculate integral over $x$ and perform summation with help of Riemann $\xi$-function. Finally, we find

$$S^{(4)} = -\sum_q \frac{7\zeta(3)}{64\pi^2 T_c^2} |\Delta|^4 \tag{A.10}$$

## A.3 Calculations in the $g^2$ order

To find the second order equations for the level population and for the anomalous average we have to derive the equations for the expectation values appearing on the right hand sides of (5.1) and (5.2). These equations for the expectation values are the following:

$$\frac{d}{dT'}\left\langle a^{\dagger}_{p_1+}a^{\dagger}_{p_2-}a_{p_3-}a_{q_2+}\right\rangle = i\left\langle [H_{int}(T'), a^{\dagger}_{p_1+}a^{\dagger}_{p_2-}a_{p_3-}a_{q_2+}]\right\rangle,$$

$$\frac{d}{dT'}\left\langle a^{\dagger}_{q_1+}a^{\dagger}_{p_2-}a_{p_3-}a_{p_4+}\right\rangle = i\left\langle [H_{int}(T'), a^{\dagger}_{q_1+}a^{\dagger}_{p_2-}a_{p_3-}a_{p_4+}]\right\rangle,$$

$$\frac{d}{dT'}\left\langle a_{q_1+}a^{\dagger}_{p_1+}a_{p_3-}a_{p_4+}\right\rangle = i\left\langle [H_{int}(T'), a_{q_1+}a^{\dagger}_{p_1+}a_{p_3-}a_{p_4+}]\right\rangle,$$

$$\frac{d}{dT'}\left\langle a^{\dagger}_{p_2-}a_{p_3-}a_{p_4+}a_{q_2-}\right\rangle = i\left\langle [H_{int}(T'), a^{\dagger}_{p_2-}a_{p_3-}a_{p_4+}a_{q_2-}]\right\rangle.$$

Using Wick's theorem and then integrating all impulses that possible we obtain:

$$\left\langle [a^{\dagger}_{k_1+}a^{\dagger}_{k_2-}a_{k_3-}a_{k_4+}, a^{\dagger}_{p_1+}a^{\dagger}_{p_2-}a_{p_3-}a_{q_2+}]\right\rangle =$$

$$= \delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{p}_3+\vec{q}_2)[-\chi^*_{k_1}n_{k_2}(n_{k_4}-1)\chi_{q_2}+\chi^*_{k_1}n_{k_2}(n_{k_4})\chi_{q_2}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{k}_3+\vec{q}_2)[\chi^*_{k_1}n_{k_2}(1-n_{k_4})\chi_{q_2}-\chi^*_{k_1}(1-n_{k_2})n_{k_4}\chi_{q_2}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_1-\vec{q}_2)\delta^3(\vec{k}_3+\vec{k}_4)[\chi^*_{k_1}n_{k_2}n_{p_1}\chi_{k_4}-\chi^*_{k_1}(n_{k_2}-1)n_{p_1}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{q}_2)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{p}_3-\vec{p}_2)[n_{k_1}n_{k_2}(1-n_{k_4})n_{p_2}-(1-n_{k_1})n_{k_2}n_{k_4}n_{p_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{q}_2)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{p}_1+\vec{p}_2)[n_{k_1}n_{k_2}\chi_{k_4}\chi^*_{p_1}+(1-n_{k_1})n_{k_2}\chi_{k_4}\chi^*_{p_1}]-$$

$$-\delta^3(\vec{k}_1-\vec{q}_2)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{k}_3-\vec{p}_2)[n_{k_1}\chi^*_{p_1}\chi_{k_4}(1-n_{k_3})-(1-n_{k_1})\chi^*_{p_1}\chi_{k_4}n_{k_3}]-$$

$$-\delta^3(\vec{k}_1-\vec{q}_2)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_3-\vec{p}_2)[n_{k_1}\chi^*_{p_1}\chi_{k_4}n_{p_2}+(1-n_{k_1})\chi^*_{p_1}\chi_{k_4}n_{p_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{q}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_1+\vec{p}_2)[-n_{k_1}n_{k_2}\chi_{k_4}\chi^*_{p_1}+(1-n_{k_1})(1-n_{k_2})\chi_{k_4}\chi^*_{p_1}]+$$

$$+\delta^3(\vec{k}_1-\vec{q}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{k}_3-\vec{p}_2)[n_{k_1}n_{k_2}(1-n_{p_1})(1-n_{k_3})-(1-n_{k_1})(1-n_{k_2})n_{p_1}n_{k_3}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3-\vec{p}_2)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{p}_3+\vec{q}_2)[\chi^*_{k_1}(1-n_{k_3})(1-n_{p_1})-\chi^*_{k_1}n_{k_3}n_{p_1}\chi_{q_2}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3-\vec{p}_2)\delta^3(\vec{p}_1-\vec{q}_2)\delta^3(\vec{p}_3+\vec{k}_4)[-\chi^*_{k_1}(1-n_{k_3})n_{p_1}\chi_{k_4}-\chi^*_{k_1}n_{k_3}n_{p_1}\chi_{k_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3+\vec{q}_2)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{p}_3-\vec{p}_2)[-\chi^*_{k_1}\chi_{q_2}(1-n_{p_1})n_{p_2}-\chi^*_{k_1}\chi_{q_2}n_{p_1}n_{p_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{p}_2-\vec{k}_3)\delta^3(\vec{p}_3+\vec{q}_2)[n_{k_1}\chi^*_{p_1}(n_{p_2}-1)\chi_{q_2}-n_{k_1}\chi^*_{p_1}n_{p_2}\chi_{q_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_1+\vec{p}_2)\delta^3(\vec{k}_3+\vec{q}_2)[n_{k_1}n_{k_2}\chi^*_{p_1}\chi_{q_2}+n_{k_1}(1-n_{k_2})\chi^*_{p_1}\chi_{q_2}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_1-\vec{q}_2)\delta^3(\vec{k}_3-\vec{p}_2)[n_{k_1}n_{k_2}n_{q_2}(1-n_{p_2})-n_{k_1}(1-n_{k_2})n_{q_2}n_{p_2}]$$

$$\left\langle [a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+}, a^\dagger_{q_1+}a^\dagger_{p_2-}a_{p_3-}a_{p_4+}] \right\rangle =$$

$$= -\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{q}_1-\vec{k}_4)\delta^3(\vec{p}_3+\vec{p}_4)[\chi^*_{k_1}n_{k_2}(n_{k_4}-1)\chi_{p_4}-\chi^*_{k_1}n_{k_2}(n_{k_4})\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{q}_1-\vec{k}_4)\delta^3(\vec{k}_3+\vec{p}_4)[\chi^*_{k_1}n_{k_2}(1-n_{k_4})\chi_{p_4}-\chi^*_{k_1}(1-n_{k_2})n_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{q}_1-\vec{p}_4)\delta^3(\vec{k}_3+\vec{k}_4)[\chi^*_{k_1}n_{k_2}n_{q_1}\chi_{k_4}-\chi^*_{k_1}(n_{k_2}-1)n_{q_1}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{q}_1-\vec{k}_4)\delta^3(\vec{p}_3-\vec{p}_2)[n_{k_1}n_{k_2}(1-n_{k_4})n_{p_2}-(1-n_{k_1})n_{k_2}n_{k_4}n_{p_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{q}_1+\vec{p}_2)[n_{k_1}n_{k_2}\chi_{k_4}\chi^*_{q_1}+(1-n_{k_1})n_{k_2}\chi_{k_4}\chi^*_{q_1}]-$$

$$-\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2+\vec{q}_1)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{k}_3-\vec{p}_2)[n_{k_1}\chi^*_{q_1}\chi_{k_4}(1-n_{k_3})-(1-n_{k_1})\chi^*_{q_1}\chi_{k_4}n_{k_3}]-$$

$$-\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2+\vec{q}_1)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_3-\vec{p}_2)[n_{k_1}\chi^*_{q_1}\chi_{k_4}n_{p_2}+(1-n_{k_1})\chi^*_{q_1}\chi_{k_4}n_{p_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{q}_1+\vec{p}_2)[-n_{k_1}n_{k_2}\chi_{k_4}\chi^*_{q_1}+(1-n_{k_1})(1-n_{k_2})\chi_{k_4}\chi^*_{q_1}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{q}_1-\vec{k}_4)\delta^3(\vec{k}_3-\vec{p}_2)[n_{k_1}n_{k_2}(1-n_{q_1})(1-n_{k_3})-(1-n_{k_1})(1-n_{k_2})n_{q_1}n_{k_3}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3-\vec{p}_2)\delta^3(\vec{q}_1-\vec{k}_4)\delta^3(\vec{p}_3+\vec{p}_4)[\chi^*_{k_1}(1-n_{k_3})(1-n_{q_1})-\chi^*_{k_1}n_{k_3}n_{q_1}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3-\vec{p}_2)\delta^3(\vec{q}_1-\vec{p}_4)\delta^3(\vec{p}_3+\vec{k}_4)[-\chi^*_{k_1}(1-n_{k_3})n_{q_1}\chi_{k_4}-\chi^*_{k_1}n_{k_3}n_{q_1}\chi_{k_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{q}_1-\vec{k}_4)\delta^3(\vec{p}_3-\vec{p}_2)[-\chi^*_{k_1}\chi_{p_4}(1-n_{q_1})n_{p_2}-\chi^*_{k_1}\chi_{p_4}n_{q_1}n_{p_2}]-$$

$$-\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2+\vec{q}_1)\delta^3(\vec{p}_2-\vec{k}_3)\delta^3(\vec{p}_3+\vec{p}_4)[n_{k_1}\chi^*_{q_1}(n_{p_2}-1)\chi_{p_4}-n_{k_1}\chi^*_{q_1}n_{p_2}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{q}_1+\vec{p}_2)\delta^3(\vec{k}_3+\vec{p}_4)[n_{k_1}n_{k_2}\chi^*_{q_1}\chi_{p_4}+n_{k_1}(1-n_{k_2})\chi^*_{q_1}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{q}_1-\vec{p}_4)\delta^3(\vec{k}_3-\vec{p}_2)[n_{k_1}n_{k_2}n_{p_4}(1-n_{p_2})-n_{k_1}(1-n_{k_2})n_{p_4}n_{p_2}]$$

$$\left\langle [a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+}, a_{q_1+}a^\dagger_{p_1+}a_{p_3-}a_{p_4+}]\right\rangle =$$

$$= \delta^3(\vec{k}_1-\vec{q}_1)\delta^3(\vec{k}_3-\vec{k}_2)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{p}_3+\vec{p}_4)[n_{q_1}n_{k_2}(1-n_{k_4})\chi_{p_4}-(1-n_{q_1})n_{k_2}n_{k_4}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{q}_1)\delta^3(\vec{k}_3-\vec{k}_2)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{p}_1-\vec{p}_4)[n_{q_1}n_{k_2}\chi_{k_4}n_{p_1}+(1-n_{q_1})n_{k_2}\chi_{k_4}n_{p_1}]-$$

$$-\delta^3(\vec{k}_1-\vec{q}_1)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{k}_3+\vec{p}_4)[-n_{q_1}\chi^*_{p_1}\chi_{p_3}\chi_{p_4}-(1-n_{q_1})\chi^*_{p_1}\chi_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1-\vec{q}_1)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{p}_3+\vec{p}_4)\delta^3(\vec{k}_3+\vec{k}_4)[n_{q_1}\chi^*_{p_1}\chi_{p_3}\chi_{k_4}+(1-n_{q_1})\chi^*_{p_1}\chi_{p_4}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{q}_1)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_1-\vec{p}_4)[-n_{q_1}n_{k_2}\chi_{k_3}n_{p_1}+(1-n_{q_1})(1-n_{k_2})\chi_{k_3}n_{p_1}]+$$

$$+\delta^3(\vec{k}_1-\vec{q}_1)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{k}_3+\vec{p}_4)[-n_{q_1}n_{k_2}(1-n_{p_1})\chi_{p_4}-(1-n_{q_1})(1-n_{k_2})n_{p_1}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_1-\vec{k}_4)\delta^3(\vec{p}_3+\vec{q}_1)[-n_{k_1}n_{k_2}(1-n_{k_4})\chi_{q_1}+(1-n_{k_1})n_{k_2}n_{k_4}\chi_{q_1}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_1-\vec{q}_1)\delta^3(\vec{k}_4+\vec{p}_3)[n_{k_1}n_{k_2}(1-n_{q_1})\chi_{k_4}+(1-n_{k_1})n_{k_2}(1-n_{q_1})\chi_{k_4}]-$$

$$-\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_3+\vec{q}_1)[-n_{k_1}\chi^*_{p_1}\chi_{k_4}\chi_{q_1}-(1-n_{k_1})\chi^*_{p_1}\chi_{k_4}\chi_{q_1}]-$$

$$-\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2+\vec{p}_1)\delta^3(\vec{p}_3+\vec{k}_4)\delta^3(\vec{k}_3+\vec{q}_1)[n_{k_1}\chi^*_{p_1}\chi_{k_4}\chi_{q_1}+(1-n_{k_1})\chi^*_{p_1}\chi_{k_4}\chi_{q_1}]-$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_1-\vec{q}_1)[-n_{k_1}n_{k_2}\chi_{k_4}(1-n_{q_1})+(1-n_{k_1})(1-n_{k_2})\chi_{k_4}(1-n_{q_1})]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{q}_1)\delta^3(\vec{p}_1-\vec{k}_4)[n_{k_1}n_{k_2}\chi_{q_1}(1-n_{k_4})+(1-n_{k_1})(1-n_{k_2})\chi_{q_1}n_{k_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3+\vec{q}_1)\delta^3(\vec{k}_4-\vec{p}_1)\delta^3(\vec{p}_3+\vec{p}_4)[-\chi^*_{k_1}\chi_{q_1}(1-n_{k_4})\chi_{p_4}-\chi^*_{k_1}\chi_{q_1}n_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{k}_4-\vec{p}_1)\delta^3(\vec{p}_3+\vec{q}_1)[\chi^*_{k_1}\chi_{p_4}(1-n_{k_4})\chi_{q_1}+\chi^*_{k_1}\chi_{p_4}n_{k_4}\chi_{q_1}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{q}_1)\delta^3(\vec{p}_1-\vec{p}_4)[n_{k_1}n_{k_2}\chi_{q_1}n_{p_1}+n_{k_1}(1-n_{k_2})\chi_{q_1}n_{p_1}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{p}_1-\vec{q}_1)[n_{k_1}n_{k_2}\chi_{p_4}(1-n_{q_1})+n_{k_1}(1-n_{k_2})\chi_{p_4}(1-n_{q_1})]$$

$$\left\langle [a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+}, a^\dagger_{p_2-}a_{p_3-}a_{p_4+}a_{q_2-}]\right\rangle =$$

$$= -\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{q}_2+\vec{p}_4)[\chi^*_{k_1}n_{k_2}\chi_{k_4}\chi_{p_4}+\chi^*_{k_1}(1-n_{k_2})\chi_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{q}_2+\vec{k}_4)[-\chi^*_{k_1}n_{k_2}\chi_{k_4}\chi_{p_4}-\chi^*_{k_1}(1-n_{k_2})\chi_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{q}_2)\delta^3(\vec{k}_3+\vec{k}_4)\delta^3(\vec{p}_3+\vec{p}_4)[-\chi^*_{k_1}n_{k_2}\chi_{k_4}\chi_{p_4}-\chi^*_{k_1}(1-n_{k_2})\chi_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{p}_2)\delta^3(\vec{k}_2-\vec{q}_2)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{p}_3+\vec{k}_4)[\chi^*_{k_1}n_{k_2}\chi_{k_4}\chi_{p_4}+\chi^*_{k_1}(1-n_{k_2})\chi_{k_4}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_2-\vec{p}_3)\delta^3(\vec{q}_2+\vec{k}_4)[-n_{k_1}n_{k_2}n_{p_2}\chi_{k_4}-(1-n_{k_1})n_{k_2}n_{p_2}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{k}_3)\delta^3(\vec{p}_2-\vec{q}_2)\delta^3(\vec{p}_3+\vec{k}_4)[n_{k_1}n_{k_2}n_{p_2}\chi_{k_4}+(1-n_{k_1})n_{k_2}n_{p_2}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_2-\vec{q}_2)\delta^3(\vec{k}_3+\vec{k}_4)[n_{k_1}n_{k_2}n_{p_2}\chi_{k_4}-(1-n_{k_1})(1-n_{k_2})n_{p_2}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{p}_2-\vec{k}_3)\delta^3(\vec{q}_2+\vec{k}_4)[n_{k_1}n_{k_2}(1-n_{p_2})\chi_{k_4}+(1-n_{k_1})(1-n_{k_2})n_{p_2}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{q}_2)\delta^3(\vec{p}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{k}_4)[n_{k_1}n_{k_2}n_{p_2}\chi_{k_4}-(1-n_{k_1})(1-n_{k_2})n_{p_2}\chi_{k_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{p}_4)\delta^3(\vec{k}_2-\vec{q}_2)\delta^3(\vec{p}_2-\vec{k}_3)\delta^3(\vec{p}_3+\vec{k}_4)[n_{k_1}n_{k_2}(1-n_{p_2})\chi_{k_4}+(1-n_{k_1})(1-n_{k_2})n_{p_2}\chi_{k_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{p}_2-\vec{k}_3)\delta^3(\vec{q}_2+\vec{p}_4)\delta^3(\vec{p}_3+\vec{k}_4)[-\chi^*_{k_1}(1-n_{p_2})\chi_{k_4}\chi_{p_4}-\chi^*_{k_1}n_{p_2}\chi_{k_4}\chi_{p_4}]-$$

$$-\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{p}_2-\vec{k}_3)\delta^3(\vec{p}_3+\vec{p}_4)\delta^3(\vec{q}_2+\vec{k}_4)[\chi^*_{k_1}(1-n_{p_2})\chi_{k_4}\chi_{p_4}+\chi^*_{k_1}n_{p_2}\chi_{k_4}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3-\vec{p}_2)\delta^3(\vec{p}_4+\vec{q}_2)[n_{k_1}n_{k_2}(1-n_{p_2})\chi_{p_4}-n_{k_1}(1-n_{k_2})n_{p_2}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{p}_3)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{p}_2-\vec{q}_2)[n_{k_1}n_{k_2}n_{p_2}\chi_{p_4}+n_{k_1}(1-n_{k_2})n_{p_2}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{q}_2)\delta^3(\vec{k}_3-\vec{p}_2)\delta^3(\vec{p}_4+\vec{p}_3)[-n_{k_1}n_{k_2}(1-n_{p_2})\chi_{p_4}+n_{k_1}(1-n_{k_2})n_{p_2}\chi_{p_4}]+$$

$$+\delta^3(\vec{k}_1-\vec{k}_4)\delta^3(\vec{k}_2-\vec{q}_2)\delta^3(\vec{k}_3+\vec{p}_4)\delta^3(\vec{p}_2-\vec{p}_3)[-n_{k_1}n_{k_2}n_{p_2}\chi_{p_4}-n_{k_1}(1-n_{k_2})n_{p_2}\chi_{p_4}]$$

1.4

$$\int \frac{d^3p_1..d^3p_4d^3k_1..d^3k_4}{(2\pi)^12}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})t}\cdot\delta^3(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)e^{i(\epsilon_{k_1}+\epsilon_{k_2}-\epsilon_{k_3}-\epsilon_{k_4})t'}\cdot$$

$$\cdot\delta^3(\vec{q}_1-\vec{p}_4)\left\langle[a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+},a^\dagger_{p_1+}a^\dagger_{p_2-}a_{p_3-}a_{q_2+}]\right\rangle=$$

$$=-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{k}-\vec{l})e^{i(2\epsilon_k-2\epsilon_q)t}[-\chi^*_kn_p\chi_q]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{k}-\vec{l}+\vec{q}_2)e^{i(\epsilon_p-\epsilon_q)(t+t')}e^{i(\epsilon_l-\epsilon_k)(t-t')}[\chi^*_pn_k(1-n_l)\chi_q-\chi^*_p(1-n_k)n_l\chi_q]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{l})e^{i(2\epsilon_p-2\epsilon_k)t'}[\chi^*_pn_q\chi_k]+$$

$$+\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{q})[n_qn_k(1-n_l)n_p-(1-n_q)n_kn_ln_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{q}_1)e^{i(2\epsilon_p-2\epsilon_q)t}[n_k\chi_l\chi^*_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{k}+\vec{l}-\vec{q})e^{i(\epsilon_p-\epsilon_l)(t+t')}e^{i(\epsilon_k-\epsilon_q)(t-t')}[n_q\chi^*_p\chi_l(1-n_k)-(1-n_q)\chi^*_p\chi_ln_k]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{q})e^{i(2\epsilon_q-2\epsilon_k)t'}[n_q\chi^*_l\chi_kn_p+(1-n_q)\chi^*_l\chi_kn_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{q})e^{i(2\epsilon_p-2\epsilon_q)t}e^{i(2\epsilon_q-2\epsilon_k)t'}[-n_qn_l\chi_k\chi^*_p+(1-n_q)(1-n_l)\chi_k\chi^*_p]+$$

$$+\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{k}+\vec{p}-\vec{l}-\vec{q})e^{i(\epsilon_p-\epsilon_q)(t-t')}e^{i(\epsilon_k-\epsilon_l)(t-t')}[n_qn_l(1-n_p)(1-n_k)-(1-n_q)(1-n_l)n_pn_k]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{l})e^{i(2\epsilon_p-2\epsilon_q)t}e^{i(2\epsilon_k-2\epsilon_p)t'}[\chi^*_k(1-n_l)(1-n_p)\chi_q-\chi^*_kn_ln_p\chi_q]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{l})e^{i(2\epsilon_k-2\epsilon_p)t'}[-\chi^*_k(1-n_p)n_q\chi_l-\chi^*_kn_pn_q\chi_l]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{q})e^{i(2\epsilon_k-2\epsilon_q)t'}[-\chi^*_k\chi_q(1-n_l)n_p-\chi^*_k\chi_{q_2}n_ln_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{l})e^{i(2\epsilon_p-2\epsilon_q)t}[n_k\chi^*_p(n_l-1)\chi_q-n_k\chi^*_pn_l\chi_{q_2}]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{q})e^{i(2\epsilon_p-2\epsilon_q)t}[n_kn_l\chi^*_p\chi_q+n_k(1-n_l)\chi^*_p\chi_{q_2}]+$$

$$+\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}-\vec{p})[n_kn_ln_q(1-n_p)-n_k(1-n_l)n_qn_p]$$

2.4

$$\int \frac{d^3p_1..d^3p_4d^3k_1..d^3k_4}{(2\pi)^12}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})t}\cdot\delta^3(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)e^{i(\epsilon_{k_1}+\epsilon_{k_2}-\epsilon_{k_3}-\epsilon_{k_4})t'}\cdot$$

$$\cdot\delta^3(\vec{p}_1-\vec{q}_2)\left\langle[a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+},a^\dagger_{q_1+}a^\dagger_{p_2-}a_{p_3-}a_{p_4+}]\right\rangle=$$

$$=-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}[\chi^*_l n_k(n_q-1)\chi_p-\chi^*_l n_k(n_q)\chi_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}-\vec{k}-\vec{l}-\vec{p})e^{i(\epsilon_k-\epsilon_p)(t+t')}e^{i(\epsilon_q-\epsilon_l)(t-t')}[\chi^*_k n_l(1-n_q)\chi_p-\chi^*_k(1-n_l)n_q\chi_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{k}+\vec{l})e^{i(2\epsilon_k-2\epsilon_p)t'}[\chi^*_k n_l n_q\chi_p-\chi^*_k(n_l-1)n_q\chi_p]+$$

$$+\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}-\vec{l})[n_l n_k(1-n_q)n_p-(1-n_l)n_k n_q n_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{k}-\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t}[n_k n_p\chi_l\chi^*_q+(1-n_k)n_p\chi_l\chi^*_q]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})e^{i(\epsilon_q-\epsilon_p)(t+t')}e^{i(\epsilon_k-\epsilon_l)(t-t')}[n_l\chi^*_q\chi_p(1-n_k)-(1-n_l)\chi^*_q\chi_p n_k]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t'}[n_l\chi^*_q\chi_k n_p+(1-n_l)\chi^*_q\chi_k n_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{k}+\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t}e^{i(2\epsilon_k-2\epsilon_p)t'}[-n_k n_l\chi_p\chi^*_q+(1-n_k)(1-n_l)\chi_p\chi^*_q]+$$

$$+\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}+\vec{l}-\vec{p}-\vec{k})e^{i(\epsilon_q+\epsilon_l-\epsilon_k-\epsilon_p)(t-t')}[n_k n_p(1-n_q)(1-n_l)-(1-n_k)(1-n_p)n_q n_l]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}e^{i(2\epsilon_k-2\epsilon_q)t'}[\chi^*_k(1-n_l)(1-n_q)\chi_p-\chi^*_k n_l n_q\chi_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}+\vec{l})e^{i(2\epsilon_k-2\epsilon_p)t'}[-\chi^*_k(1-n_p)n_q\chi_l-\chi^*_k n_p n_q\chi_l]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)e^{i(2\epsilon_k-2\epsilon_q)t'}\delta^3(\vec{q}-\vec{l})[-\chi^*_k\chi_l(1-n_q)n_p-\chi^*_k\chi_l n_q n_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{q}+\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}[n_k\chi^*_q(n_l-1)\chi_p-n_k\chi^*_q n_l\chi_p]-$$

$$-\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{l}+\vec{p})e^{i(2\epsilon_q-2\epsilon_l)t}[n_k n_l\chi^*_q\chi_p+n_k(1-n_l)\chi^*_q\chi_p]+$$

$$+\int\frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1-\vec{q}_2)\delta^3(\vec{p}-\vec{l})[n_k n_l n_q(1-n_p)-n_k(1-n_l)n_q n_p]$$

3.4

$$\int \frac{d^3p_1..d^3p_4d^3k_1..d^3k_4}{(2\pi)^12}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})t}\cdot\delta^3(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)e^{i(\epsilon_{k_1}+\epsilon_{k_2}-\epsilon_{k_3}-\epsilon_{k_4})t'}\cdot$$

$$\cdot\delta^3(\vec{p}_2-\vec{q}_2)\left\langle[a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+},a_{q_1+}a^\dagger_{p_1+}a_{p_3-}a_{p_4+}]\right\rangle=$$

$$=+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}[n_qn_k(1-n_l)\chi_p-(1-n_q)n_kn_l\chi_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{l}-\vec{q})[n_qn_k\chi_ln_p+(1-n_q)n_k\chi_ln_p]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{p}+\vec{l}+\vec{k})e^{i(\epsilon_q+\epsilon_p-\epsilon_k-\epsilon_l)(t+t')}[-n_q\chi^*_p\chi_k\chi_l-(1-n_q)\chi^*_p\chi_k\chi_l]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}e^{i(2\epsilon_q-2\epsilon_k)t'}[n_q\chi^*_l\chi_p\chi_k+(1-n_q)\chi^*_l\chi_p\chi_k]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t'}[-n_qn_l\chi_kn_p+(1-n_q)(1-n_l)\chi_kn_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{k}+\vec{p}-\vec{l})e^{i(\epsilon_l-\epsilon_k)(t-t')}e^{i(\epsilon_q-\epsilon_p)(t+t')}[-n_qn_k(1-n_l)\chi_p-(1-n_q)(1-n_k)n_l\chi_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{l}-\vec{p})[-n_pn_k(1-n_l)\chi_q+(1-n_p)n_kn_l\chi_q]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{l}-\vec{k})e^{i(2\epsilon_q-2\epsilon_k)t}[n_kn_p(1-n_q)\chi_l+(1-n_k)n_p(1-n_q)\chi_l]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{p}-\vec{l})e^{i(2\epsilon_l-2\epsilon_k)t'}[-n_l\chi_p\chi^*_k\chi_q-(1-n_l)\chi_p\chi^*_k\chi_q]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(-\vec{q}+\vec{p}+\vec{k}-\vec{l})e^{i(\epsilon_p-\epsilon_k)(t+t')}e^{i(\epsilon_q-\epsilon_l)(t-t')}[n_l\chi^*_p\chi_k\chi_q+(1-n_l)\chi^*_p\chi_k\chi_q]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{l}+\vec{k})e^{i(2\epsilon_q-2\epsilon_k)t}e^{i(2\epsilon_k-2\epsilon_p)t'}[-n_kn_l\chi_p(1-n_q)+(1-n_k)(1-n_l)\chi_p(1-n_q)]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(-\vec{q}+\vec{l}-\vec{k}-\vec{p})e^{i(\epsilon_l+\epsilon_q-\epsilon_k-\epsilon_p)(t-t')}[n_kn_p\chi_q(1-n_l)+(1-n_k)(1-n_p)\chi_qn_l]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)delta(\vec{l}-\vec{q})e^{i(2\epsilon_q-2\epsilon_p)t}e^{i(2\epsilon_k-2\epsilon_q)t^{'}}[-\chi_k^*\chi_q(1-n_l)\chi_p-\chi_k^*\chi_q n_l\chi_p]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{l}+\vec{p})e^{i(2\epsilon_k-2\epsilon_l)t^{'}}[\chi_k^*\chi_p(1-n_l)\chi_q+\chi_k^*\chi_p n_l\chi_q]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{l})[n_k n_l\chi_q n_p+n_k(1-n_l)\chi_q n_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)e^{i(2\epsilon_q-2\epsilon_l)t}\delta^3(\vec{l}+\vec{p})[n_k n_l\chi_p(1-n_q)+n_k(1-n_l)\chi_p(1-n_q)]$$

4.4

$$\int \frac{d^3p_1..d^3p_4d^3k_1..d^3k_4}{(2\pi)^12}\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)e^{i(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})t}\cdot\delta^3(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)e^{i(\epsilon_{k_1}+\epsilon_{k_2}-\epsilon_{k_3}-\epsilon_{k_4})t'}\cdot$$

$$\cdot\delta^3(\vec{q}_1-\vec{p}_1)\left\langle[a^\dagger_{k_1+}a^\dagger_{k_2-}a_{k_3-}a_{k_4+},a^\dagger_{p_2-}a_{p_3-}a_{p_4+}a_{q_2-}]\right\rangle$$

$$=-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{k}+\vec{l})e^{i(2\epsilon_k-2\epsilon_p)t'}[\chi^*_k n_l\chi_p\chi_q+\chi^*_k(1-n_l)\chi_p\chi_q]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{k}-\vec{l}-\vec{p})e^{i(\epsilon_k-\epsilon_p)(t+t')}e^{i(\epsilon_q-\epsilon_l)(t-t')}[-\chi^*_k n_l\chi_q\chi_p-\chi^*_k(1-n_l)\chi_q\chi_p]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}e^{i(2\epsilon_q-2\epsilon_k)t'}[-\chi^*_l n_q\chi_k\chi_p-\chi^*_l(1-n_q)\chi_k\chi_p]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{k}+\vec{l}-\vec{p})e^{i(\epsilon_q+\epsilon_k-\epsilon_l-\epsilon_p)(t+t')}[\chi^*_k n_q\chi_l\chi_p+\chi^*_k(1-n_q)\chi_l\chi_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{l})[-n_l n_k n_p\chi_q-(1-n_l)n_k n_p\chi_q]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{k}-\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t}[n_k n_p n_q\chi_l+(1-n_k)n_p n_q\chi_l]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{k}+\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t}e^{i(2\epsilon_k-2\epsilon_p)t'}[n_k n_l n_q\chi_p-(1-n_k)(1-n_l)n_q\chi_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{l}-\vec{p}-\vec{k})e^{i(\epsilon_q+\epsilon_l-\epsilon_k-\epsilon_p)(t-t')}[n_k n_p(1-n_l)\chi_q+(1-n_k)(1-n_p)n_l\chi_q]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}-\vec{l})e^{i(2\epsilon_q-2\epsilon_k)t'}[n_l n_q n_p\chi_k-(1-n_l)(1-n_q)n_p\chi_k]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{p}+\vec{l}-\vec{k})e^{i(\epsilon_q-\epsilon_l)(t+t')}e^{i(\epsilon_p-\epsilon_k)(t-t')}[n_k n_q(1-n_p)\chi_l+(1-n_k)(1-n_q)n_p\chi_l]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{p}+\vec{l})e^{i(2\epsilon_k-2\epsilon_p)t'}[-\chi^*_k(1-n_p)\chi_l\chi_q-\chi^*_k n_p\chi_l\chi_q]-$$

$$-\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}e^{i(2\epsilon_k-2\epsilon_q)t'}[\chi^*_k(1-n_l)\chi_q\chi_p+\chi^*_k n_l\chi_q\chi_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{l}-\vec{p})[n_k n_p(1-n_l)\chi_q-n_k(1-n_p)n_l\chi_q]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{p}-\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}[n_kn_pn_q\chi_l+n_k(1-n_p)n_q\chi_l]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{l})e^{i(2\epsilon_q-2\epsilon_p)t}[-n_kn_q(1-n_l)\chi_p+n_k(1-n_q)n_l\chi_p]+$$

$$+\int \frac{d^3pd^3kd^3l}{(2\pi)^9}\delta^3(\vec{q}_1+\vec{q}_2)\delta^3(\vec{q}+\vec{l})[-n_kn_qn_p\chi_l-n_k(1-n_q)n_p\chi_l]$$

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